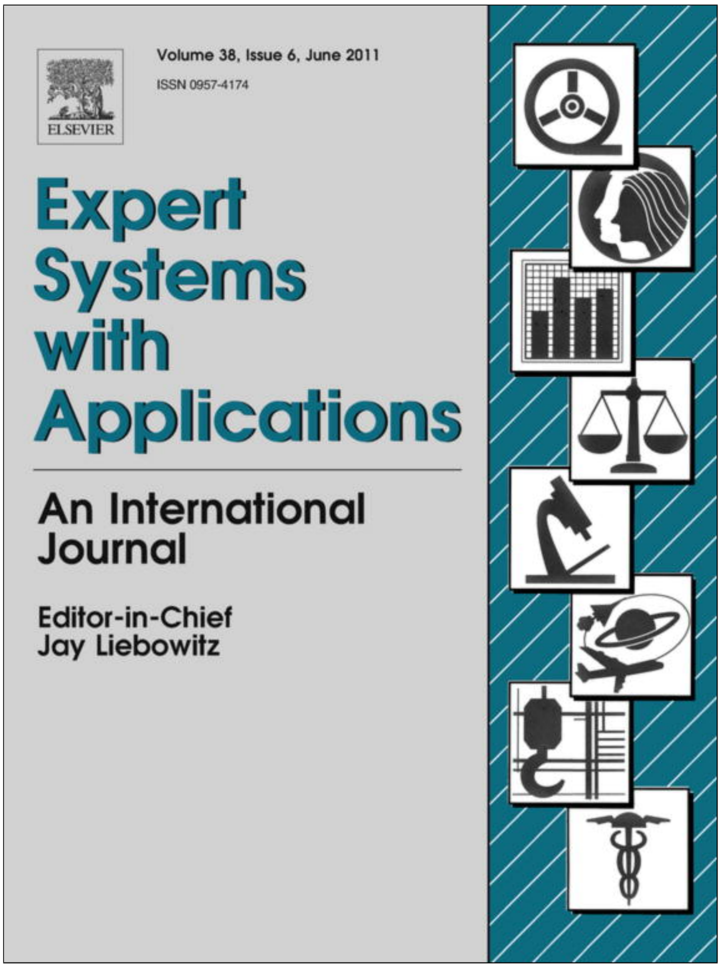
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# An evolutionary approach for multi-objective optimization of the integrated location–inventory distribution network problem in vendor-managed inventory

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ABSTRACT

Vendor-managed inventory (VMI) is one of the emerging solutions for improving the supply chain efficiency. It gives the supplier the responsibility to monitor and decide the inventory replenishments of their customers. In this paper, an integrated location–inventory distribution network problem which integrates the effects of facility location, distribution, and inventory issues is formulated under the VMI setup. We presented a Multi-Objective Location–Inventory Problem (MOLIP) model and investigated the possibility of a multi-objective evolutionary algorithm based on the Non-dominated Sorting Genetic Algorithm (NSGA2) for solving MOLIP. To assess the performance of our approach, we conduct computational experiments with certain criteria. The potential of the proposed approach is demonstrated by comparing to a well-known multi-objective evolutionary algorithm. Computational results have presented promise solutions for different sizes of problems and proved to be an innovative and efficient approach for many difficult-to-solve problems.



## 1. Introduction

Recently, two generic strategies for supply chain design have emerged: efficiency and responsiveness. Efficiency aims to reduce operational costs; responsiveness, on the other hand, is designed to react quickly to satisfy customer demands. A crucial question in the supply chain is the design of distribution networks and the identification of facility locations. Ballou and Masters (1993) put forward four strategic planning areas in the design of a distribution network system, as shown in Fig. 1. The first issue deals with customer service levels. The second one deals the placement of facilities and demand assignments made to them. The third deals with inventory decisions and policies that involve inventory control. The fourth deals with transportation decisions of how transport modes are selected, utilized, and controlled. All four of these areas are inter-related and the customer service level is determined by the other three decision areas. There are practical challenges for firms when they try to simultaneously reduce operating costs (for efficiency) and customer service (for responsiveness). In traditional supply chain network design, the optimization focus is often placed on minimizing cost and maximizing profit as a single objective. However, very few distribution network systems should be considered as intrinsically single objective problems. It is not always desirable to reduce costs if this results in a degraded level of customer service. Thus, it is necessary to set up a multi-objective network design problem.

Research on integrated location–inventory distribution network systems is relatively new. Jayaraman (1998) developed an integrated model which jointly examined the effects of facility location, transportation modes, and inventory-related issues. However, Jayaraman's study did not contain any demand and capacity restrictions. Erlebacher and Meller (2000) formulated an analytical joint location–inventory model with a highly nonlinear objective function to maintain acceptable service while minimizing operating, inventory and transportation costs. Nozick and Turnquist (2001) proposed a joint location–inventory model to consider both cost and service responsiveness trade-offs based on an uncapacitated facility location problem. Miranda and Garrido (2004) studied a MINLP model to incorporate inventory decisions into typical facility location models. They solved the distribution network problem by incorporating a stochastic demand and risk pooling phenomenon. Sabri and Beamon (2000) presented an integrated multi-objective, multi-product, multi-echelon model that simultaneously addresses strategic and operational planning decisions by developing an integrated two sub-module model which includes cost, fill rates, and flexibility. Gaur and Ravindran (2006) studied a bi-criteria optimization model to represent the inventory aggregation problem under risk pooling, finding out the tradeoffs in costs and responsiveness.

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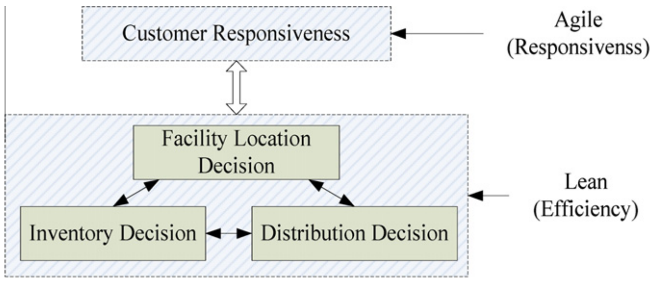


**Fig. 1.** Four strategic planning issues in distribution network design.

Recently, Daskin, Coullard, and Shen (2002) and Shen, Coullard, and Daskin (2003) introduced a joint location–inventory model with risk pooling (LMRP) that incorporates inventory costs at distribution centres (DCs) into location problems. LMRP solved the problem in two special cases: deterministic demand and Poisson demand. It assumed direct shipments from DCs to buyers which extended the uncapacitated fixed-charge problems to incorporate inventory decisions at the DCs. The uncapacitated assumption at DCs is usually not the case in practice. Shu, Teo, and Shen (2005) solved LMRP with general stochastic demand. Shen and Daskin (2005) extended the LMRP model to include the customer service component and proposed a nonlinear multi-objective model including both cost and service objectives. In contrast to LMRP and its variants that consider inventory cost only at the DC level, Teo and Shu (2004) and Romeijn, Shu, and Teo (2007) proposed a warehouse-retailer network design problem in which both DCs and retailers carried inventory. These are actually the two major streams of integrated distribution network design problems.

Our model builds upon the initial LMRP model but with some differences. First, a capacitated version of a similar model is established. Second, to make an original contribution, the proposed model incorporates two extra performance metrics corresponding to customer service. With these considerations, we present a capacitated Multi-Objective Location–Inventory Problem (MOLIP) which results in a Mixed-Integer Non-Linear Programming (MINLP) formulation. Some noteworthy innovative research aspects that are incorporated in our research include: (i) *Multi-Objective Location–Inventory Problem*. Very few studies have addressed this problem; (ii) *multi-objective evolutionary algorithms* (MOEAs). Most previous works have focused on traditional optimization techniques, but few have performed these techniques successfully and efficiently. In contrast, MOEAs have been successfully developed for various optimization problems, creating potential for the proposed MOLIP.

This study is organized as follows: Section 2 describes our research problem and details the model formulation. Section 3 proposes a hybrid evolutionary algorithm with a heuristic procedure for MOLIP. Section 4 illustrates our experimental results including (i) the computational results of a *base-case* problem (ii) scenario analysis (iii) computational evaluation of the proposed algorithm for MOLIP. Finally, conclusions and suggestions for the direction of future research are provided in Section 5.

## 2. Designing an integrated location–inventory distribution network model

In this section, we present a mathematical model which provides the foundation for our research.

### 2.1. Problem description

#### 2.1.1. VMI coordination mechanism

Vendor-managed inventory (VMI) is one of the most widely discussed coordination mechanisms for improving multi-firm supply chain efficiency. Evidence has shown that VMI can improve supply chain performance by decreasing inventory costs for the supplier and buyer and improving customer service levels, such as reduced order cycle times and higher fill rates (Waller, Johnson, & Davis 1999). Fig. 2 indicates the system diagram of a VMI system includes its incurred material and information flows. Since the supplier is responsible for managing the inventories at the buyer's DC, including ordering and inventory holding, the supplier ought to receive the information about demand directly from the market. Since the supplier determines ordering instead of receiving orders from buyers, there is no information flow of the buyer's orders in the VMI system.

The main feature of VMI indicates the centralized system within, with which the supplier as a sole decision maker decides the order quantity based on information available from both buyers and suppliers to minimize the total cost of the whole supply chain system. The supplier has full authority over inventory management at the buyer's DC to pay all costs associated with the supplier's production cost, both the buyer's and the supplier's ordering cost, the inventory holding cost and distribution cost. The supplier monitors, manages and replenishes the inventory of the buyer. Thus, the decisions on order replenishment quantity and order shipping are given to the supplier in the VMI system, rather than to the buyer as in tradition systems. Fig. 3 presents the operational cost structure between the partners in the VMI system. The proposed model is mainly based this cost structure.

#### 2.1.2. Overview of our research problem

In general, suppliers and distributors route their products through DCs. In practice, there are many cases in which each supplier has its own set of DCs. Consider a distribution network configuration problem where a single supplier and DCs are to be established to distribute various products to a set of buyers and both the DCs and buyers are geographically dispersed in a region. In this problem, each buyer experiences demands for a variety of products, which are provided by the supplier. A set of DCs must be located in the distribution network from a list of potential sites. The DCs act as intermediate facilities between the supplier and the buyers and facilitate the shipment of products between the two echelons. The supplier wishes to decide the supply chain distribution network for its products such as to determine the subsets of DCs to be opened and to design a distribution network strategy that will satisfy all capacity and demand requirements for the products imposed by the buyers.

However, our problem jointly considers both *strategic* and *tactical* decisions in the supply chain system. The strategic decision involves the *location* problem, which determines the number and the locations of DCs and assigns buyers to DCs, whereas the tactical decision deals with the *inventory* problem which determines the levels of safety stock inventory at DCs to provide certain service levels to buyers. The integrated problem is called a *location–inventory* distribution network problem. The centralized inventory policy is considered under the vender managed inventory (VMI) mode (Waller et al., 1999) which refers to the holding safety stocks aggregated at DCs. This inclusion acquires especial relevance in the presence of high inventory holding costs and high variability of demands. Fig. 4 shows the overall schematic diagram of the hierarchy of the model considered in our study.

### 2.2. Model assumptions and notations

Basic assumptions are used when modeling our problem. It is assumed that all the products are produced by a single supplier and one specific product for a buyer should be shipped from a single DC. Reverse flows, in-transit inventory, and pipeline inventory are not considered. All the buyers' demands are uncertain and the

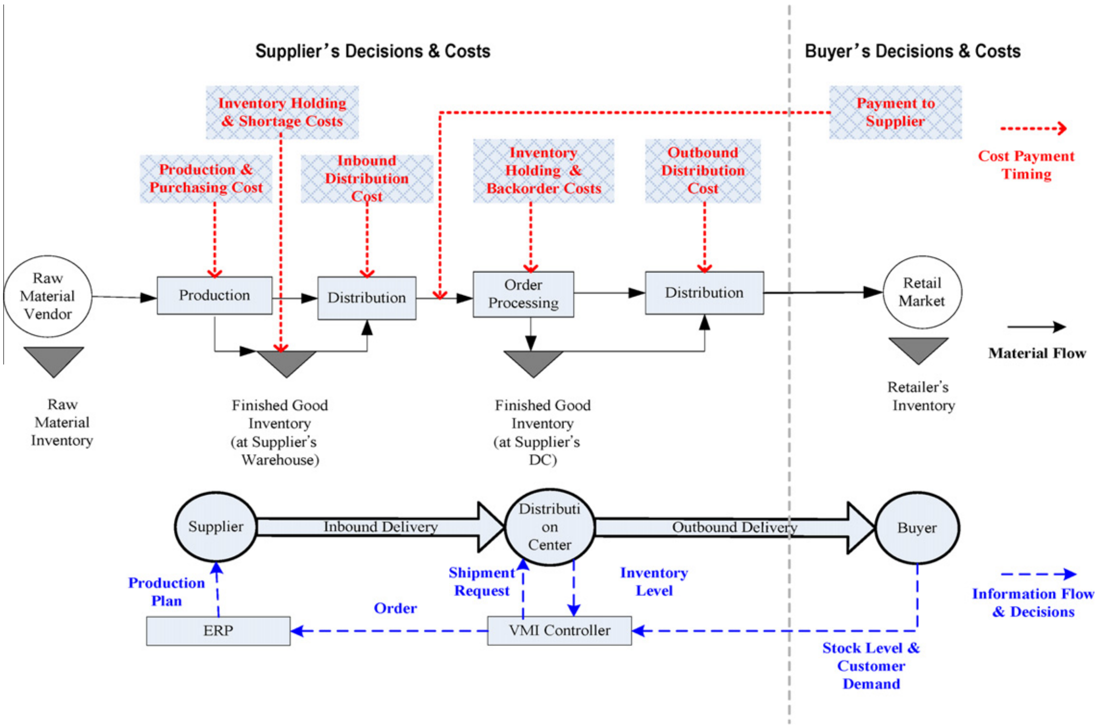


**Fig. 2.** System diagram of VMI system.

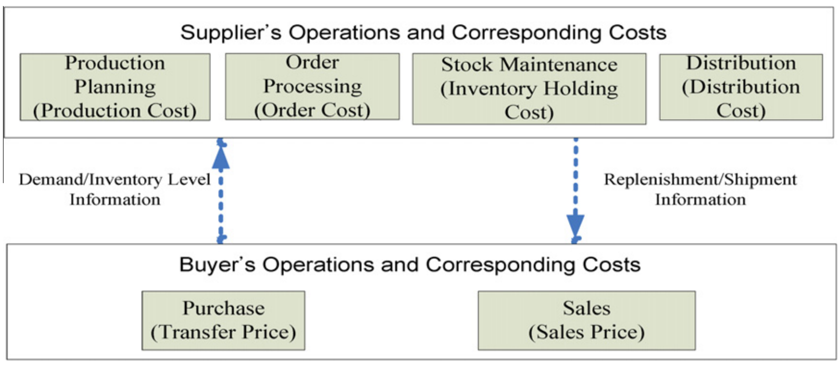


**Fig. 3.** Cost structure of VMI system.

storage capacities of the supplier are unlimited but are capacitated at open DCs. More assumptions will be stated when we illustrate the mathematical model. Here, the mathematical notations used in the model are described as follows.

*Indices:* $i$ is an index set for buyers ($i \in I$). $j$ is an index set of potential DCs ($j \in J$). $k$ is an index set for product classifications ($k \in K$).

*Decision variables:* $Q_{wj}^{k}$ is the aggregate economic order quantity for DC $j$ for product k shipped from the supplier. $Y_j = 1$ if DC $j$ is open (=0, otherwise). $X_{ji}^{k} = 1$ if DC $j$ serves buyer $i$ for shipping product $k$ (=0, otherwise).

*Model parameters:* $u_j$ is the capacity volume of DC $j$. $d_{ik}$ is the average daily demands for product $k$ of buyer $i$. $\sigma_{ik}$ is the standard deviation of daily demands for product $k$ of buyer $i$. $\zeta_j^k$ is the average lead time (daily) for product $k$ to be shipped to DC $j$ from the supplier. $\psi$ is the number of days per year. $f_j$ is the fixed annual facility operating cost of opening a DC $j$. $h_j^k$ is the aggregated inventory holding unit cost per unit time (annually) at DC $j$ for product $k$. $tc_{ji}^{k}$ is the unit variable transportation cost for shipping product $k$ from DC $j$ to buyer $i$. $rc_j^k$ is the unit variable production and transportation cost for shipping product k from the supplier to DC $j$. $D_{wj}^{k}$ is the expected annual demand for product $k$ through DC $j$. $D_{\max}$ is the maximal covering distance, that is, buyers within this distance from an open DC are considered well satisfied.

## 2.3. Mathematical model

To begin modeling this problem, let us assume for a moment that the assignment of buyers to a DC is known a priori and that all the products are produced by a single supplier. We assume that the daily demand for product $k$ at each buyer $i$ is independent and normally distributed, i.e., $N(d_{ik}, \sigma_{ik}^2)$. Furthermore, at any site of DC $j$, we assume a continuous review inventory policy $(Q_j, r_j)$ to meet a stochastic demand pattern. Also, we consider that the supplier takes an average lead time $\zeta_j^k$ (in days) for shipping product $k$ from the supplier to DC $j$ so as to fulfill an order. From basic inventory theory (Eppen, 1979), we know that if the demands at each buyer are uncorrelated, then the aggregate demand for product $k$ during lead time $\zeta_j^k$ at the DC $j$ is normally distributed with a mean of $\zeta_j^k d_{wj}^k$,

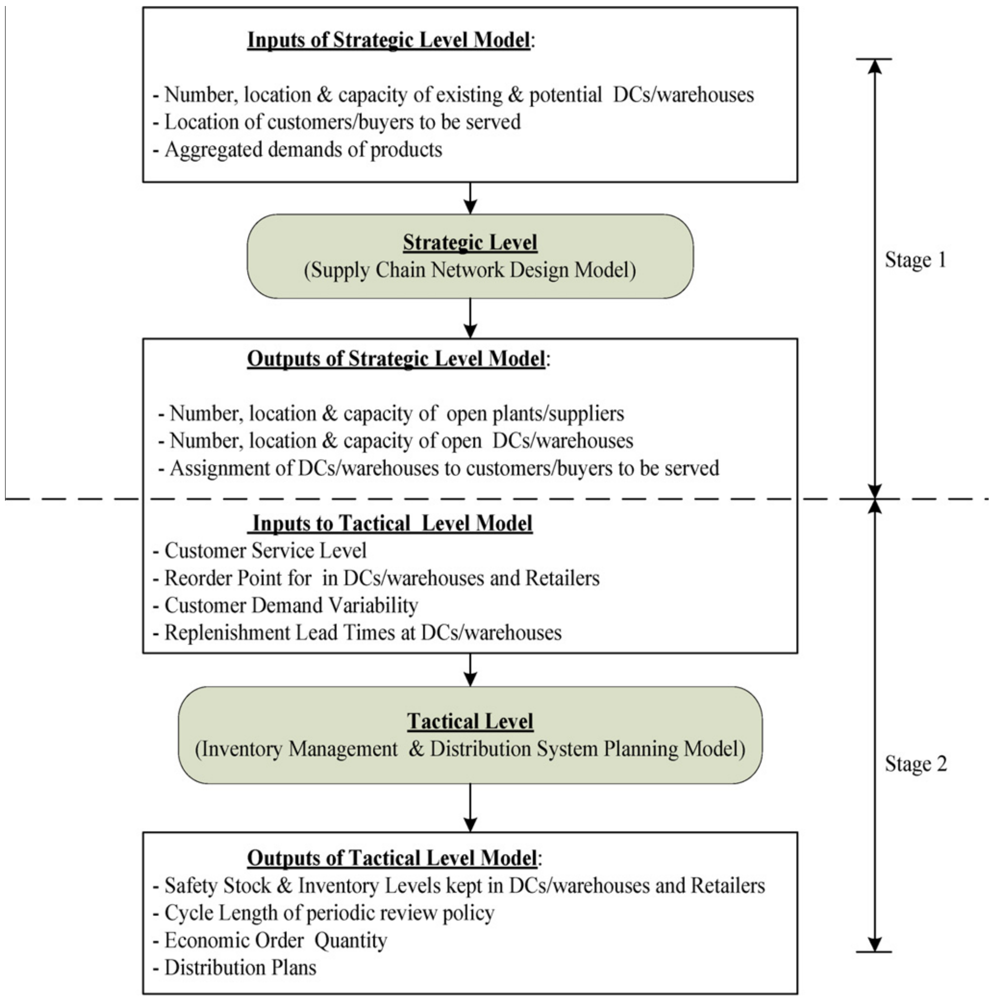


**Fig. 4.** Overview of the integrated strategic and tactical planning model.

where $d_{wj}^k = \sum_{i\in I} d_{ik} X_{ji}^k$, and a variance of $\zeta_j^k \sum_{i\in I} \sigma_{ik}^2 X_{ji}^k$. Let us consider the centralized supply chain system under the vendor managed inventory (VMI) mode, which refers to aggregating the safety stock pooled at different DCs. Then, the total amount of safety stock for product $k$ at DC $j$ with risk pooling is $z_{1-\alpha}\sqrt{\zeta_j^k \sum_{i\in I} \sigma_{ik}^2 X_{ji}^k}$, where $1-\alpha$ refers to the *level of service* for the system and $z_{1-\alpha}$ is the standard normal value with $P(z \leqslant z_{1-\alpha}) = 1-\alpha$.

In the proposed model, the total cost is based on the cost structure of the VMI system in Fig. 3 and is decomposed into the following items: (i) *facility cost*, which is the cost of setting up DCs, (ii) *transportation cost*, which is the cost of transporting products from the supplier to the buyers via specific DCs, (iii) *operating cost*, which is the cost of running DCs, (iv) *cycle stock cost*, which is the cost of ordering and maintaining inventory at DCs, and (v) *safety stock cost*, which is the cost of holding sufficient inventory at DCs in order to provide a specific service level to their buyers. The total cost $Z_1$ is represented as follows:

$$Z_1 = \sum_{j\in J} f_j \cdot Y_j + \psi \sum_{k\in K}\sum_{j\in J}\sum_{i\in I}\left(rc_j^k + tc_{ji}^k\right)\cdot d_{ik}\cdot X_{ji}^k + \sum_{k\in K}\cdot\sum_{j\in J} o_j^k \cdot \frac{\psi\cdot d_{wj}^k}{Q_{wj}^k} + \sum_{k\in K}\sum_{j\in J} h_j^k \cdot \frac{Q_{wj}^k}{2}\cdot Y_j^k + \sum_{k\in K}\sum_{j\in J} h_j^k \cdot z_{1-\alpha}\sqrt{\zeta_j^k \sum_{i\in I}\sigma_{ik}^2 \cdot X_{ji}^k} \quad (1)$$

Based on $Z_1$, the optimal order quantity $Q_{wj}^{k^*}$ for product $k$ at each DC $j$ can be obtained by differentiating Eq. (1) in terms of $Q_{wj}^k$, each DC $j$ and each product $k$ equal zero to minimize the total supply chain cost. We can obtain $Q_{wj}^{k^*} = \sqrt{2\cdot o_j^k \cdot D_{wj}^k / h_j^k}$ for $\forall$ open DC $j$, $\forall\, k$. In this case, there is not any capacity constraint for the order quantities $Q_{wj}^k$ since we assume the storage capacity at the supplier is unlimited. Thus, replacing $Q_{wj}^{k^*}$ in the third and fourth terms of $Z_1$ in Eq. (1), we can obtain a non-linear cost function of $Z_1$. As follows, we propose an innovative mathematical model for the Multi-Objective Location-Inventory Problem (MOLIP).

$$\text{Min } Z_1 = \sum_{k\in K}\sum_{j\in J} f_j \cdot Y_j^k + \sum_{k\in K}\sum_{j\in J}\sum_{i\in I} \Psi_{ji}^k \cdot X_{ji}^k + \sum_{k\in K}\cdot\sum_{j\in J}\left(\Gamma_j^k \sqrt{\sum_{i\in I} D_{ik}\cdot X_{ji}^k}\right) + \sum_{k\in K}\sum_{j\in J} h_j^k \cdot \sqrt{\sum_{i\in I} \Lambda_{ji}^k \cdot X_{ji}^k} \quad (2)$$

$$\text{Max } Z_2 = \left(\sum_{k\in K}\sum_{i\in I} d_{ik} X_{ji}^k\right) \Big/ \sum_{k\in K}\sum_{i\in I} d_{ik} \quad (3)$$

$$\text{Max } Z_3 = \left(\sum_{k\in K}\sum_{i\in I} d_{ik} \sum_{j\in \tau_i} X_{ji}^k\right) \Big/ \sum_{k\in K}\sum_{i\in I} d_{ik} X_{ji}^k \quad (4)$$

$$\text{s.t.} \quad \sum_{j\in J} X_{ji}^k \le 1 \quad \forall i \in I; \quad \forall k \in K \quad (5)$$

$$X_{ji}^k \le Y_j \quad \forall i \in I; \quad \forall j \in J; \quad \forall k \in K \tag{6}$$

$$\sum_{k\in K}\sum_{i\in I} d_{ik}\cdot X_{ji}^k + \sum_{k\in K}\sqrt{\sum_{i\in I}\Lambda_{ji}^k\cdot X_{ji}^k} \le u_j Y_j \quad \forall j \in J \tag{7}$$

$$X_{ji}^k \in \{0,1\} \quad Y_j \in \{0,1\}, \quad \forall i \in I; \quad \forall j \in J; \quad \forall k \in K \tag{8}$$

where $\Psi_{ji}^k = \psi\cdot\left(rc_j^k + tc_{ji}^k\right)\cdot d_{ik} \quad \Gamma_j^k = \sqrt{2\cdot o_j^k\cdot h_j^k}$

$$D_{ik} = \psi\cdot d_{ik} \quad \Lambda_{ji}^k = (z_{1-\alpha})^2\cdot\zeta_j^k\cdot\sigma_{ik}^2$$

Eqs. (2)–(4) give the objectives. While Eq. (2) of $Z_1$ is to minimize the total cost, Eq. (3) of $Z_2$ and Eq. (4) of $Z_3$ give the objectives by referring to the maximization of customer service by two performance measurements: (i) *volume fill rate* (VFR), defined as the satisfied fraction of total demands without shortage; (ii) *responsiveness level* (RL), the percentage of fulfilled demand volume within specified coverage distance $D_{\max}$. Eq. (5) restricts a buyer to be served by a single DC if possible. Eq. (6) stipulates that buyers can only be assigned to open DCs. Eq. (7) indicates the maximal capacity restrictions on the opened DCs to enable the capability of holding sufficient inventory for every product that flows through the DC, and also part of safety stock so as to maintain the specified service level. Eq. (8) determines binary constraints. The proposed MOLIP model does not only determine the DC locations, the assignment of buyers to DCs, but also finds out endogenously both the optimal order quantities and safety-stock levels at DCs. Since two of the three objective functions ($Z_1$ and $Z_3$) are nonlinear, the formulation results in an intractable multi-objective MINLP model.

## 3. Problem solving methodology

### 3.1. Multi-objective evolutionary algorithms

Evolutionary algorithms (Michalewicz, 1996) are known to be efficient-solving and easily-adaptive for these problems. On the contrary, traditional methods have failed to provide good solutions to such problems (e.g. MINLP). Recently, since the pioneering work by Schaffer (1985), multiobjective evolutionary algorithms (MOEAs) have prevailed. There are many efficient MOEAs (Deb, Pratap, Agarwal, & Meyarivan, 2002; Fonseca & Fleming, 1993; Knowles & Corne, 2000; Srinivas & Deb, 1994; Zitzler & Thiele, 1999) that are able to find Pareto optimal solutions as well as widely distributed solutions. However, the well-known MOEA called the *elitist non-dominated sorting* GA or NSGA-II (Deb et al., 2002) is one of the most successful approaches. In our research, a hybrid genetic algorithm which incorporates the NSGA-II and a heuristic assignment procedure is proposed to optimize the proposed MOLIP.

### 3.2. NSGAII-based evolutionary algorithm

Multiobjective optimization problems give rise to a set of Pareto-optimal solutions, none of which can be said to be better than the other for all objectives. Unlike most traditional optimization approaches, evolutionary algorithms (EAs) work with a population of solutions and are thus likely candidates for finding multiple Pareto-optimal solutions (Coello Coello, 1999). There are primarily two goals to be achieved in multiobjective EAs (MOEA): (i) convergence to a Pareto-optimal set, and (ii) maintenance of population diversity in a Pareto-optimal set. Most MOEAs work with the concept of *domination*. When a problem has more than one objective functions (say $f_j$, $j = 1, 2, \ldots, m$ and $m > 1$), any two solutions $x_1$ and $x_2$ can exhibit one of two possibilities, one dominates the other or neither dominates the other. One solution $x_1$ is said to *dominate* the other solution $x_2$ if both the following conditions are true: (i) the solution $x_1$ is not worse than $x_2$ for all objectives; (ii) the solution $x_1$ is strictly better than $x_2$ in at least one objective. If any of the above conditions are violated, the solution $x_1$ does not *dominate* $x_2$.

NSGA-II (Deb et al., 2002) is one of the best techniques for generating Pareto frontiers in MOEAs. First of all, for each solution in the population, one has to determine how many solutions dominate it and the set of solutions to which it dominates. Then, it ranks all solutions to form non-dominated fronts according to a *non-dominated sorting* process, hence, classifying the chromosomes into several fronts of non-dominated solutions. To allow for diversification, NSGA-II also estimates the solution density surrounding a particular solution in the population by computing a *crowding distance* operator. During selection, a *crowded-comparison* operator considering both the *non-domination rank* of an individual and its *crowding distance* is used to select the offspring, without losing good solutions (*elitism*). Whereas, crossover and mutation operators remain, as usual.

A NSGAII-based evolutionary approach for MOLIP is proposed, as shown in Table 1. This algorithm starts by generating a random population $P(1)$ of size $L$. For each chromosome in $P(1)$ the algorithm evaluates its cost and coverage using the encoded solution expressions. Then, the algorithm applies non-dominated sorting of $P(1)$ and assigns each chromosome to the front to which it belongs. Next, the algorithm applies binary tournament selection (to form the crossover pool), crossover, and mutation operators to generate the children population $C(1)$ of size $L$. Once initialized, the main body of the algorithm repeats for T generations. The algorithm applies *non-dominated sorting* to $R(t)$, resulting in a population from the union of parents $P(t)$ and children $C(t)$. The algorithm obtains the next generation population $P(t+1)$ after selecting the $L$ chromosomes from the first fronts of $R(t)$. Next, it applies binary tournament selection, crossover, and mutation operators to generate the children $C(t+1)$. Too make it easy to understand, the algorithm is graphically represented in Fig. 5.

During the selection process of the next generation, chromosome fitness depends on the evaluation of the decoded solution in the objective functions and its comparison with other chromosomes. The *non-domination sorting* updates a tentative set of Pareto optimal solutions by ranking a population according to non-domination. After that, each individual $p$ in the population is given two attributes: (i) *non-domination rank* in the optimization objectives ($p$.rank); (ii) *local crowding distance* in the objectives space directions ($p$.distance). If both chromosomes are at the same non-domination rank, the one with fewer chromosomes around in the front

**Table 1**
NAGAII-based evolutionary approach.

1: Randomly generate $P(1)$
2: Evaluate $P(1)$
3: Non-dominated sort $P(1)$
4: Generate $C(1)$ form $P(1)$, apply binary tournament selection, crossover, and mutation
5: Evaluate $C(1)$
6: **while** $t \leqslant T$ **do**
7: $R(t) = P(t) \cup C(t)$
8: Nondominated sort $R(t)$
9: Sort $R(t)$ using $\geqq_n$ {see Definition 1}
10: Select $P(t+1)$ from the first L chromosome of $R(t)$
11: Generate $C(t+1)$ from $P(t+1)$, apply binary tournament selection, crossover, and mutation
12: Mutate $C(t+1)$
13: Evaluate $C(t+1)$
14: $t \leftarrow t+1$
15: **end while**

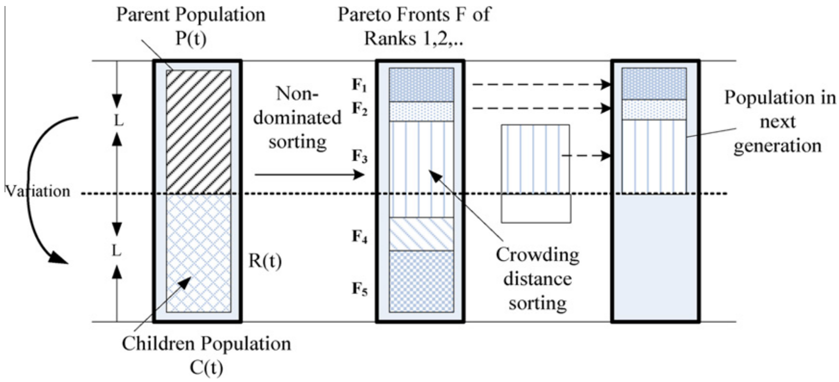


**Fig. 5.** Graphical representation of the NSGA-II algorithm.

is preferred. Thus, a partial order ($\geqq_n$) defined in Definition 1 is used to decide which of the two chromosomes is fitter. Definition 1 states that a higher non-domination level chromosome is always preferred. If chromosomes are at the same level, the one with fewer chromosomes around the front is preferred.

**Definition 1.** Let $p, q \in R(t)$ be chromosomes in population $R(t)$. $p$ is said to be fitter than $q$ ($p \geqq_n q$), either if ($p$.rank < $q$.rank) or (($p$.rank = $q$.rank) and ($p$.distance > $q$.distance)).

### 3.3. Solution encoding

Each solution of the MOLIP is encoded in a binary string of length m = |I|, where the $j$-th position (bit) indicates if DC $j$ is open (value of 1) or closed (value of 0). This binary encoding only considers if a given DC $j$ is open or closed (variables $Y_j$). A MOLIP solution also involves the assignment of buyers to open DCs (variables $X_{ji}^k$). This assignment is performed by a procedure that tries to minimize cost without deteriorating coverage and capacity. Since the capacity constraints in MOLIP limit the amount of buyer's demands that can be assigned to candidate DCs, a *greedy* heuristic is used to fulfill the buyer-DC assignments. First of all, the buyers are sorted in the descending order of their demand flows. After that, according to the sorted order, they are assigned to a specific DC according to the following rules:

*Rule 1:* For each buyer $i$, if the buyer $i$ is covered (i.e., there are DCs within a distance of $D_{\max}$), it is assigned to the DC with sufficient capacity (if one exists) which can serve it with the minimal difference between the remaining capacity of an open DC $j$ and the demand flow of the buyer $i$ through DC $j$. That is, the DC assignment attempts to be as full as possible.

*Rule 2:* If the buyer $i$ cannot be covered or there is no successful assignment from the coverage set, it is then assigned to the candidate DC (with sufficient capacity) that increases the total cost by the least amount, regardless of its distance to the DC, if possible.

## 4. Model applications and experimental results

### 4.1. A base-line problem and its computational results

There are no MOLIP instances in the public domain, nor are any available in previous studies to serve for benchmarking. For this reason, a base-line problem was developed by taking the size of a Gamma.com supply chain network with 15 DCs and 50 buyers as reference. The potential DC locations are randomly generated within a square of 100 distance units of width. Other model parameters are given in Table 2. For the sake of simplicity, Euclidean distance is used for measuring distribution distances. The company intended to determine the number of open DCs needed for order assignments. However, the capacity limitation of DCs affects the assignments of buyers. The managers also need to evaluate tradeoffs among three criteria: total cost (TC), volume fill rate (VFR) and responsiveness level (RL). To obtain the approximate Pareto front, we attempted to solve the specified problem using the proposed hybrid evolutionary approach. Through the GA approach, the base-line model (# of DCs = 15, # of buyers = 50) with product number (k = 2) resulted in 765 binary variables and 815 constraints.

In addition, defining a reference point is the first step in allowing the MOLIP to obtain tradeoff solutions. The reference point is a vector formed by the single-objective optimal solutions and is the best possible solution that may be obtained for a multi-objective problem. With a given reference point, the MOLIP problem can then be solved by locating the alternative(s) or decision(s) which have the minimum distance to the reference point. Thus, the problem becomes how to measure the distance to the reference point. For the MOLIP problem, the decision maker is asked to determine weights by prior knowledge of objectives once all the alternatives in the Pareto front are generated. Moreover, the reference point can be found simply by optimizing one of the original objectives at a time subjective to all constraints. Due to the incommensurability among objectives, we measure this distance by using normalized Euclidean distance between two points in $k$-dimensional vector space, $d = \left\{\sum_{t=1}^{k} w_t \left[(f_t^* - f_t)/f_t^*\right]^2\right\}^{1/2}$, where $f$ is an alternative solution in the Pareto front, $f^*$ is the reference point and $w_t$ is the relative weight for the $t$-th objective. Then, all alternatives are ranked based on the value of $d$ in descending order. The highest ranked alternative (with the minimal value of $d$) is then considered as the "optimal" solution among alternatives for the given MOLIP problem.

**Table 2**
Model parameters for the base-line problem.

| Parameters | Value |
|---|---|
| Annual cost of operating a DC $j$ | U(900, 1000) |
| Annual holding cost at DC $j$ for product $k$ | U(.2, .4) |
| Unit ordering cost at DC $j$ for product $k$ | U(8, 10) |
| Capacity of DC $j$ | U(500, 700) |
| Unit variable transportation cost | $1 |
| Unit production and shipping cost for product $k$ from the supplier to DC $j$ | U(1, 3) |
| Maximal covering distance | 25 Km |
| Lead time (daily) | U(2, 4) |
| Working days per year | 260 |
| Average daily demand for product $k$ at buyer $i$ | U(60, 80) |
| Standard deviation of daily demands | U(2, 4) |
| Standard normal value (service level = 0.95) | 1.96 |

The hybrid evolutionary approach is used for the *base-line* model. The input parameters are: population size = 100; generation number = 200; cloning = 20%; crossover rate = 80%; mutation rate varies from 5% to 10%. The approach program was coded in MATLAB. The algorithm allows the decision maker to rapidly find a set of Pareto solutions that are large in number. The decision maker requires the determination of weights by prior knowledge of objectives to generate the user-defined 'optimal' solution. As shown in Fig. 6, we illustrate a user-defined 'optimal' solution among the alternatives with equal weights of three objectives (i.e., $w_1 = w_2 = w_3 = 1/3$) in the *base-line* model with the minimal total cost of $251,112,536, the maximal volume fill rate of 71.97%, and the maximal customer responsive level of 62.15%, respectively, where nine out of 15 candidate DCs are required to open and aggregate. It is worth mentioning that most of these aggregated DCs were assigned to the buyers as close as to them as possible within the maximal coverage (within 25 kms). However, about 29.03% of buyers were unassigned ($\varnothing$), revealing the percentage of the uncovered demands which could possibly result in sales losses. Also, 37.85% of aggregated buyers ($\rightarrow$) were assigned to DCs farther than the coverage distance.

Fig. 7 shows the approximate Pareto front of the base-line problem obtained from the NSGAII-based evolutionary algorithm. To make it easy to understand the existing tradeoff between the cost and volume fill rate and responsive level, respectively, we present it as a percentage of the minimal cost instead of using it in an absolute term. As shown in Fig. 2, it is possible to increase volume fill rate (VFR) from 31.87% to 69.82% and responsiveness level (RL) from 25.39% to 63.61% if the percentage over the minimal cost increases from 204% to 400.55% (about two times) when the number of open DCs is increased from four to nine. Thus, if the decision maker's goal is to maintain volume fill rate (VFR) at a level of about 70%, compared to the current status of 31.87%, extra costs are necessary increase open DCs up to nine. The increase in DCs enhances customer's volume fill rate and also increase responsiveness level at the same time.

### 4.2. Performance evaluation

Our goal here is to evaluate the efficiency and the effectiveness of the proposed NSGAII-based evolutionary approach. We establish a set of random instances and try to keep almost all model parameters the same as the *base-case* problem. We generate problem instances of different sizes of DCs and buyers in the distribution network. In addition, various capacity and facility-cost scenarios are considered again. In this experiment, we generated four sets of problem instances (SET 1 to SET 4) representing different sizes of problem instances ranging from 15 DCs and 50 buyers, to 100

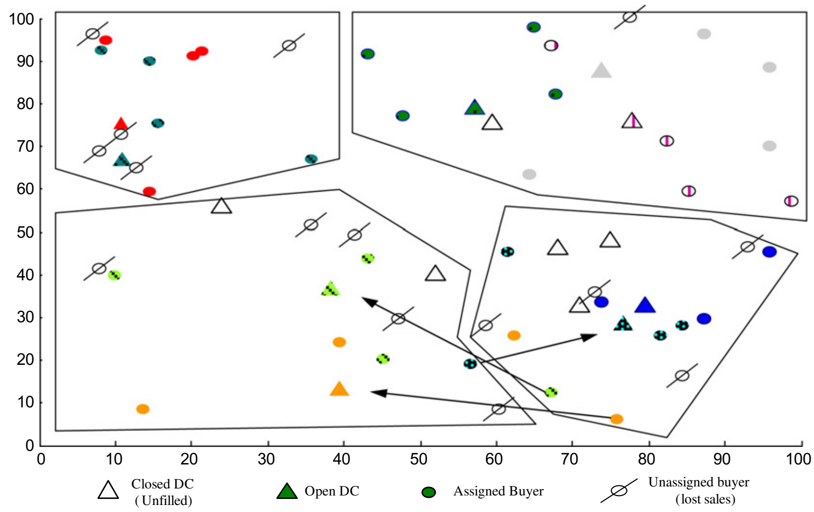


**Fig. 6.** Graphical display of the user-defined 'optimal' solution.

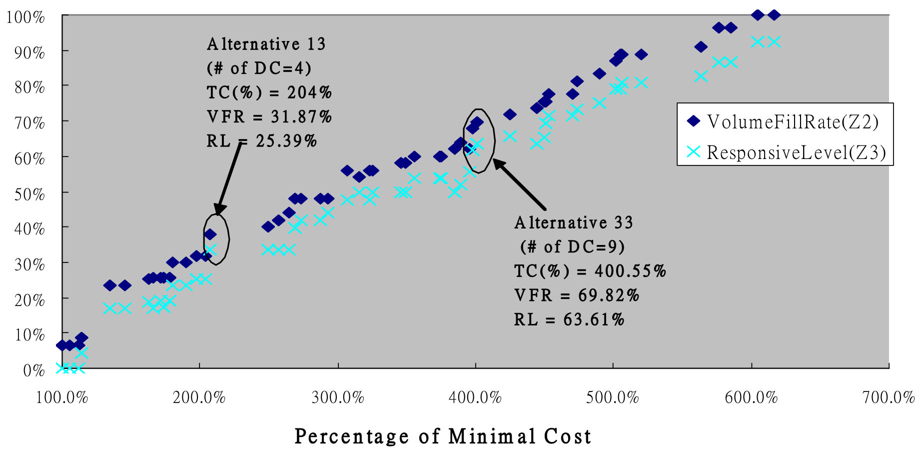


**Fig. 7.** Approximate Pareto front for the user-defined 'optimal' solution.

DCs and 500 buyers (problem sizes m_n: 15_50 (SET 1), 50_150 (SET 2), 75_300 (SET 3) and 100_500 (SET 4). Similar to the base-case problem, all these instances are randomly generated and uniformly distributed to locations within a square of 100 distance units of width for the coordinates of all DCs and buyers. However, there are two different types of facility-cost structures (F1 to F2). Instances labeled F1 and F2 represent different types of facility cost problems. There are also problem instances with three different types of DC capacity scenarios (C1 to C3). Instances labeled C1, C2 and C3 stand for different DC capacity structures corresponding to *tight*, *normal* and *excess* capacity scenarios. After combining all the possibilities of problem sizes (four types), facility cost structure (two types) and DC capacity structure (three types), we end up with 24 problem instances. Each problem instance is given a name in the following format: Am_n_(F1 to F2)_(C1 to C3). For instance, the problem instance A50_150_F2_C1 represents a problem in which there are 50 DCs with 150 buyers which are both uniformly distributed within the square area width of 100 distance units. The facility cost is expensive and its DC capacity structure is rather tight as compared to others.

SPEA2 (Zitzler & Thiele, 1999) and NSGAII (Deb et al., 2002) have been considered two of the most successful and standard evolutionary approaches among studies on MOEAs. To verify the efficacy of the algorithm, we try to make comparisons between NSGAII and SPEA2 in the MOLIP model by using the randomly generated 24 problem instances mentioned above. Ten independent runs of each problem instance were conducted for each algorithm. The final computational results of each problem instance are obtained by aggregating the approximate Pareto solutions of the 10 independent runs. Table 3 summarizes the performance results of the two algorithms considered. For each algorithm, we report on the identification of the instances, the standardized dominated-space metric $\psi$ similar to that proposed by Medaglia and Fang (2003), and the number of solutions ($|PF^*|$) found in the approximate Pareto front, respectively. Note that the dimensionless metric $\psi$ ranges between 0 and 1 and as it approaches 1, the closer the approximate and true Pareto front are to each other. In addition, we represent $|PF^*|$ in $(w, a, b)$ format, where $w$, $a$ and $b$ indicate the worst, the average and the best values in the 10 runs of the algorithms. Finally, the last two columns indicate the incremental percentage of the average execution time ($T$) of the 10 independent runs (in seconds) and the standardized *dominated-space metric* ($\psi$) to find out their relative differences between SPEA2 and NSGAII.

From Table 3, we conclude that for small instances in SET 1 ($m = 15$, $n = 50$), NSGAII obtained better results in $\psi$. Nonetheless, SPEA2 runs faster than NSGAII. This efficiency in terms of execution time is due to the fact that SPEA2 compares the current solution with the archive (i.e., one with many), as opposed NSGAII which compares many solutions with the current Pareto frontier (i.e., many with many). However, for larger instances from SET 2 ($m = 50$, $n = 150$) to SET 4 ($m = 100$, $n = 500$), NSGAII almost always outperforms SPEA2. That is, the NSGAII obtained better results than SPEA2 in all $\psi$ metrics and almost all computing times $T$ except for those instances labeled C1 corresponding to tight capacity scenarios. In addition, there are significant differences in the quality of the solutions between the two approaches in the so called difficult-to-solve instances in SET 4.

Although NSGAII spends slightly more computation time than SPEA2 for those instances labeled C1, it still outperforms SPEA2 greatly in obtaining better quality of the approximate Pareto frontiers for larger $\psi$. For example, in the A100_500_F1_C1 instance, NSGAII runs slightly slower with a difference 1.1% compared SPEA2, but favors in solution quality with the value 20.7%. The two columns of $|PF^*|$ in Table 3 indicate how many solutions are contained on the approximate Pareto front. The results in $|PF^*|$ verify that NSGAII always provides more Pareto solutions and maintains better diversity properties than SPEA2. Furthermore, NSGAII is more stable and robust in computation. For all instances, SPEA2 is highly variable in $|PF^*|$ as compared to NSGAII. That is, the gaps between the worst value $w$ and the best value $b$ of all experiments obtained by SPEA2 are much larger than for NSGAII. Thus, we may conclude that NSGAII is a reliable method that provides more robust approximate Pareto solutions. Fig. 8 illustrates the approximate Pareto frontiers obtained by the NSGAII and the SPEA2-based algorithms for the problem instance A100_500_F1_U1. For ease of understanding of the existing tradeoffs among the three objectives, including total cost (TC), volume fill rate (VFR) and responsiveness level (RL), we normalize total cost instead of using

**Table 3**
Comparisons between NSGAII and SPEAII-based approaches.

| Instances | | Execution time ($T_{SPEA}$) | $\psi_{SPEA}$ | $\lvert PF^*\rvert$ | Execution time ($T_{NSGA}$) | $\psi_{NSGA}$ | $\lvert PF^*\rvert$ | $\frac{T_{NSGA}-T_{SPEA}}{T_{SPEA}}$ (%) | $\frac{\psi_{NSGA}-\psi_{SPEA}}{\psi_{SPEA}}$ (%) |
|---|---|---|---|---|---|---|---|---|---|
| SET 1 | A15_50_F1_C1 | 4608.92 | 0.3134 | (36, 39.6, 43) | 5384.64 | 0.3296 | (65, 69.0, 73) | 16.83 | 5.15 |
| | A15_50_F1_C2 | 281.41 | 0.3196 | (36, 40.6, 44) | 1198.86 | 0.4203 | (50, 52.1, 53) | 326.02 | 31.51 |
| | A15_50_F1_C3 | 270.30 | 0.4160 | (33, 40.3, 49) | 602.26 | 0.5003 | (53, 54.8, 57) | 122.81 | 20.27 |
| | A15_50_F2_C1 | 4489.84 | 0.3020 | (37, 44.0, 48) | 4984.81 | 0.3218 | (63, 68.4, 72) | 11.02 | 6.54 |
| | A15_50_F2_C2 | 239.69 | 0.3993 | (40, 49.0, 60) | 1941.78 | 0.4954 | (58, 59.2, 61) | 710.12 | 24.07 |
| | A15_50_F2_C3 | 167.94 | 0.4526 | (46, 49.3, 54) | 360.61 | 0.5612 | (58, 64.7, 68) | 114.73 | 23.98 |
| SET 2 | A50_150_F1_C1 | 9466.70 | 0.3795 | (27, 33.7, 40) | 10425.70 | 0.4363 | (59, 63.5, 73) | 10.1 | 15.0 |
| | A50_150_F1_C2 | 524.00 | 0.6476 | (30, 35.5, 43) | 451.70 | 0.7814 | (62, 66.1, 69) | −13.8 | 20.7 |
| | A50_150_F1_C3 | 402.40 | 0.8023 | (52, 58.2, 61) | 342.90 | 0.9425 | (70, 76.6, 83) | −14.8 | 17.5 |
| | A50_150_F2_C1 | 11709.10 | 0.2853 | (37, 43.1, 49) | 12602.30 | 0.3369 | (65, 68.5, 73) | 7.6 | 18.1 |
| | A50_150_F2_C2 | 531.80 | 0.5104 | (37, 43.6, 51) | 465.20 | 0.6460 | (59, 65.1, 71) | −12.5 | 26.6 |
| | A50_150_F2_C3 | 400.40 | 0.7617 | (42, 49.2, 55) | 347.60 | 0.9437 | (68, 71.8, 74) | −13.2 | 23.9 |
| SET 3 | A75_300_F1_C1 | 21549.50 | 0.3384 | (36, 40.8, 51) | 23061.30 | 0.3878 | (63, 67.3, 70) | 7.0 | 14.6 |
| | A75_300_F1_C2 | 1600.40 | 0.7144 | (8, 16.1, 25) | 1222.60 | 0.8024 | (56, 59.0, 63) | −23.6 | 12.3 |
| | A75_300_F1_C3 | 1703.80 | 0.7772 | (6, 19.1, 28) | 1216.10 | 0.9116 | (56, 58.5, 64) | −28.6 | 17.3 |
| | A75_300_F2_C1 | 20786.70 | 0.3313 | (31, 35.9, 47) | 22287.40 | 0.4010 | (59, 64.5, 69) | 7.2 | 21.0 |
| | A75_300_F2_C2 | 1450.00 | 0.7349 | (19, 25.9, 34) | 1117.90 | 0.8487 | (59, 63.2, 66) | −22.9 | 15.5 |
| | A75_300_F2_C3 | 1557.60 | 0.7599 | (8, 18.5, 25) | 1142.20 | 0.9019 | (55, 58.9, 62) | −26.7 | 18.7 |
| SET 4 | A100_500_F1_C1 | 36946.77 | 0.3016 | (25, 34.4, 42) | 37341.60 | 0.3641 | (63, 66.6, 72) | 1.1 | 20.7 |
| | A100_500_F1_C2 | 6523.45 | 0.6496 | (9, 11.6, 17) | 5928.16 | 0.7285 | (22, 28.8, 41) | −9.1 | 12.1 |
| | A100_500_F1_C3 | 6841.60 | 0.6913 | (9, 10.6, 14) | 6072.60 | 0.7890 | (23, 26.4, 31) | −11.2 | 14.1 |
| | A100_500_F2_C1 | 40958.10 | 0.2961 | (32, 38.4, 44) | 41824.20 | 0.3538 | (63, 66.0, 71) | 2.1 | 19.5 |
| | A100_500_F2_C2 | 6661.20 | 0.6426 | (10, 13.8, 18) | 6219.60 | 0.7558 | (22, 30.5, 42) | −6.6 | 17.6 |
| | A100_500_F2_C3 | 6504.00 | 0.7056 | (8, 12.2, 14) | 5844.70 | 0.8053 | (19, 28.3, 39) | −10.1 | 14.1 |

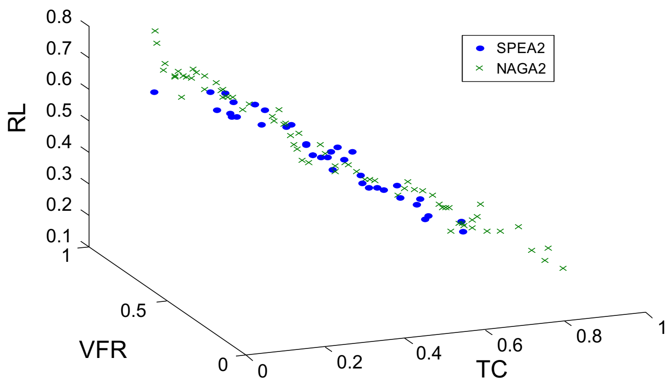


**Fig. 8.** Approximate tradeoff curves for problem instance A100_500_F1_U1.

it in absolute terms. Visually, the trade-off curves of these two approaches are very similar and partially overlapped. However, NSGAII results in the solutions covering a larger surface of the approximate Pareto solutions.

## 5. Concluding remarks and research directions

This study presented a MOLIP model initially represented as an integrated MOLIP formulation which examines the effects of facility location, distribution, and inventory issues under a vendor managed inventory (VMI) coordination mechanism. The MOLIP model is solved with a proposed hybrid evolutionary algorithm which is preliminarily based on a well-known NAGA-II evolutionary algorithm with an elitism strategy and a non-dominated sorting mechanism.

We implemented two experiments. First, we investigated the possibility of a NSGAII-based evolutionary algorithm solving the MOLIP model. Computational results revealed that the hybrid approach performed well and presented promising solutions for the MOLIP model in solving practical-size problems. Second, we compared our approach with SPEA2 to understand the efficiency among two approaches. The experiment indicates that two algorithms obtained similar approximations of the Pareto front but our approach outperformed SPEA2 in terms of the diversity quality of the approximation of the Pareto front. Moreover, SPEA2 was only efficient in terms of execution time in small or tight capacity instances. This indicates that the proposed approach could be an efficient approach for providing feasible and satisfactory solutions to large-scale difficult-to-solve problems.

In future works, we intend to adapt the proposed hybrid evolutionary algorithm to other location, inventory and distribution systems that have different characteristics or network structures. For instance, a network system may have stockpiles or inventories within the suppliers and the customer sites, and the shortage penalty needs to be considered in the overall supply chain operating cost. In addition, the inclusion of other inventory decisions would be a direction worth pursuing. Such inventory decisions could include frequency and size of the shipments from plants to the DCs and from DCs to the retailers based on different replenishment policies, and lead time in addition to safety-stock inventory in the model,. Finding ways to adapt our hybrid evolutionary algorithm into such systems is the task of future research.

Other possible research directions are to explore more competitive MOEAs or other existing optimization technologies, such as Lagrangian relaxation, particle swarm optimization, ant colony optimization, or other soft intelligent computing techniques. Comparative studies of these techniques are worth investigating in the future. In addition, some possible methods of hybridizations include the adaption of new genetic operators for integrated systems and the incorporation of other heuristic search techniques into the evolutionary algorithms, such as hill-climbing or local repair procedure.

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